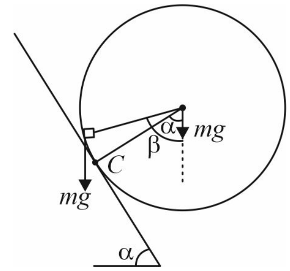
Solutions to JEE Advanced Home Practice Test -6 |JEE 2024 | Paper-1

PHYSICS



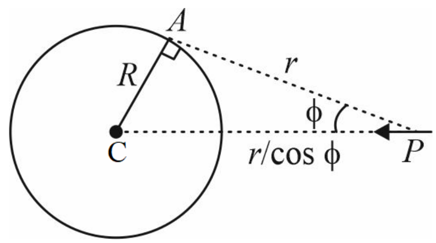
1.(B) Torque equilibrium about contact point C

$mgR\sin\alpha \le mg(R\sin\beta - R\sin\alpha)$

$\Rightarrow \quad 2\sin\alpha \le \sin\beta$

$2\sin\alpha \le 1 \;\Rightarrow\; \alpha = 30°$

2.(B) $V_A = V_C = \dfrac{kp}{(r/\cos\phi)^2}$



But $\quad V_A = V_p + V_{ind}$

Where V_p = Potential at A due to dipole &

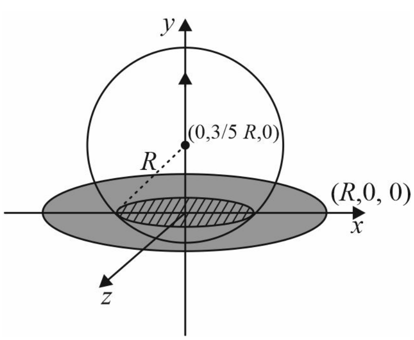
V_{ind} = Potential at A due to induced charges.

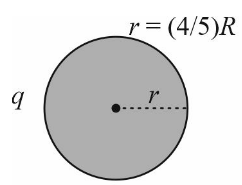
$\therefore \quad V_{ind} = \dfrac{kp}{r^2}(\cos^2\phi - 1)$

Putting value of $\phi = 30°$

$\Rightarrow \quad V_{ind} = -\dfrac{kp}{4r^2} \quad$ or $\quad V_{ind} = \dfrac{-p}{16\pi\epsilon_0 r^2}$

3.(B)



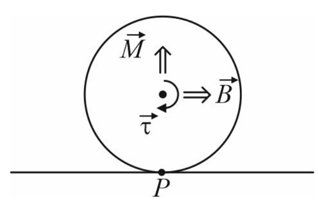


Charge on circular part

$$q = \int \sigma(r)\cdot 2\pi r\cdot dr = \int_0^{(4/5)R} 2\pi\sigma_0\left(r - \frac{r^3}{R^2}\right)dr = \frac{272}{625}(\sigma_0\pi R^2)$$

$$\phi = \frac{q}{\varepsilon_0} = \frac{272}{625}\left(\frac{\sigma_0\pi R^2}{\varepsilon_0}\right)$$

4.(A) $\dfrac{M}{L} = \dfrac{q}{2m} \Rightarrow M = \dfrac{q}{2m}\omega\dfrac{2}{3}mR^2 = \dfrac{q\omega R^2}{3}$



Torque about 'P' $= MB = I_P\alpha \Rightarrow \dfrac{q\omega R^2}{3}B = \dfrac{5}{3}mR^2\alpha$

$\therefore \quad a_{CM} = \dfrac{\alpha}{R} = \dfrac{q\omega RB}{5m}; \quad f = ma_{cm} = \dfrac{q\omega RB}{5}$

5.(A) $Y_{7D} = \frac{15}{2}\beta, \quad Y'_{3B} = \beta'$

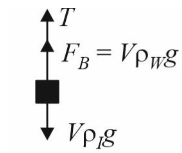
Since $Y_{8D} = Y'_{3B} \Rightarrow \frac{15}{2}\beta = 3\beta'$; $\frac{5}{2}\frac{\lambda D}{d} = \frac{\lambda D'}{d}$ or $D' = \frac{5}{2}D = 2.5D$

$\therefore$ Distance travelled $= D' - D$

$= 1.5D$ & $t = \frac{1.5D}{V}$

6.(D) Let V = volume of block

for equilibrium $\Rightarrow \quad T = V(\rho_I - \rho_W)g \quad \ldots$ (i)



FBD of Block

Let V_0 = volume of water in bucket

Then taking water + Block as system.

$$2T = V_0\rho_W g + V\rho_I g \quad \ldots \text{(ii)}$$

From (1) & (2) equation

$$2V(\rho_I - \rho_W)g = V_0\rho_W g + V\rho_I g$$

$\Rightarrow \quad V_0 = \frac{V\rho_1}{\rho_W} - 2V \Rightarrow \quad V\left[\frac{\rho_I}{\rho_W} - 2\right] \quad \Rightarrow \quad V_0 = 60 \text{ cm}^3$

7.(AC) $F = -\frac{dU}{dr} = -\frac{d}{dr}[k\ell n(r)] = -\frac{k}{r}$

$\frac{mv^2}{r} = \frac{k}{r}$; $mvr = \frac{nh}{2\pi}$; $v = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad m\sqrt{\frac{k}{m}}r = \frac{nh}{2\pi}$; $r = \frac{nh}{2\pi\sqrt{mk}}$

Again: $E = U + (KE) = k\ell nr + \frac{1}{2}m\frac{k}{m} = \frac{k}{2} + k\ell n(r)$; $E = k\left[\frac{1}{2} + \ell n\left(\frac{nh}{2\pi\sqrt{mk}}\right)\right]$

8.(BC) Cylinder absorbs energy from left face and radiate from both, so $\frac{P}{2}(1 - \cos 60°) = \sigma AT^4 + \sigma AT'^4$

$\Rightarrow \quad P = 68\,W$

Power emitted by right face reaches it through conduction.

$\therefore$ Heat current $= 1W$; $\frac{kA\Delta T}{\ell} = 1 \quad \Rightarrow \quad k = 0.057$

9.(AD) Dimensions of $\alpha = \frac{[Q]}{[I]} = [T]$; Since $-\left(\frac{tI}{\Delta V \in_0 \beta}\right)$ has to be dimension less

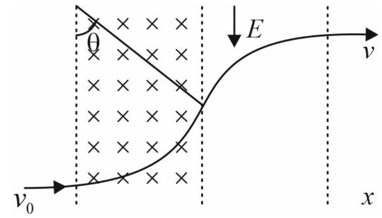
$\Rightarrow$ Dimension of β = dimension of $\frac{tI}{\Delta V \in_0} = \frac{[T][A]}{[ML^2T^{-3}A^{-1}][M^{-1}L^{-3}T^4A^2]} = [L]$

$\therefore$ Dimensions of $\left(\frac{\beta}{\alpha}\right) = LT^{-1}$ = Dimesion of velocity.

Since $C = \frac{1}{\sqrt{\mu_0 \in_0}}$ = speed of light in vacuum $\quad \therefore \quad$ option (A) = correct

Option (D) = correct

10.(BC)



$$\sin\theta = \frac{\ell}{mv_0/qB_0} = \frac{1}{2}; \qquad \theta = 30°$$

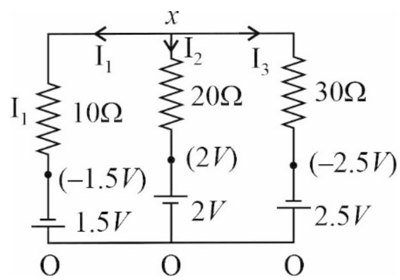
Time interval of its motion in magentic field $t_1 = \frac{\theta}{\omega} = \frac{\pi/6}{qB_0/m} = \frac{\pi m}{6qB_0}$

Time interval of its motion in electric field $t_2 = \frac{v_0 \sin 30°}{qE_0/m} = \frac{mv_0}{2qE_0}$

Further, $\ell = \frac{v_0^2 \sin 60°}{2qE_0/m} \Rightarrow \quad \frac{mv_0}{2qB_0} = \frac{mv_0^2 \sin 60°}{2qE_0} \quad \Rightarrow \quad E_0 = \frac{\sqrt{3}v_0B_0}{2}$

$\Rightarrow \quad t_2 = \frac{mv_0}{2q}\frac{2}{\sqrt{3}v_0B_0} = \frac{m}{\sqrt{3}qB_0}$; Total time $t = t_1 + t_2 = \frac{m}{6qB_0}(\pi + 2\sqrt{3})$

11.(AC) Apply KCL at node *x*.



i.e., $I_1 + I_2 + I_3 = 0$

$\Rightarrow \frac{x+1.5}{10} + \frac{x-2}{20} + \frac{x+2.5}{30} = 0$

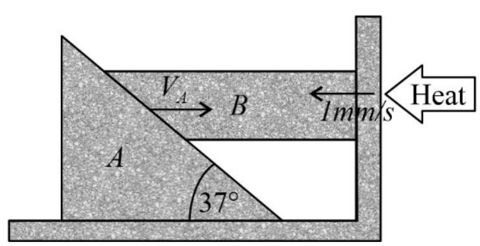
$\Rightarrow 6x + 9 + 3x - 6 + 2x + 5 = 0$

$\therefore \; x = -\frac{8}{11}V = -0.73V$

(A) $\quad I_1 = \frac{-\frac{8}{11} + \frac{3}{2}}{10}A = \frac{-16+33}{220}A = \frac{17}{220}A = 0.08A$

(C) $\quad V_B - V_A = -x = 0.73V$

12.(ABD)



$V_A = 0.5t \; mm/s^2$

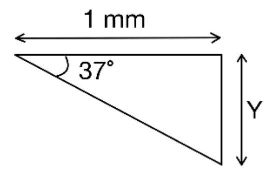
When $V_A < 1 \; mm/s$

$0.5t < 1$

$t < 2 \; sec$

Till 2 sec B falls down and stops at $t = 2\,\text{sec}$. After 2sec B starts rising up. Hence options A & B are correct.

In 2 sec, length of wax melted is $2 \times 1 = 2mm$



And wedge A moved towards wall is $S = \frac{1}{2}at^2 = \frac{1}{2} \times 0.5 \times 2^2 = 1mm$

Relative decrease in length is $1\,mm$.

Now, Y is distance travelled by B in 2 sec.

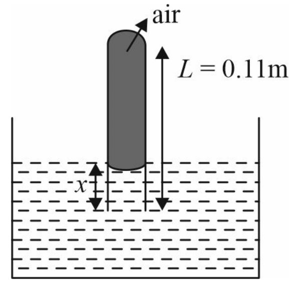
$\tan 37^0 = \frac{Y}{1} \Rightarrow Y = \frac{3}{4} = 0.75\,mm$

In next 2 sec, B goes up by same distance, hence distance travelled by block B is $1.5\,mm$ in $4\,\text{sec}$.

13.(0.25) $mgR = \mu mgs$; $\mu = \frac{R}{S} = \frac{1}{4} = 0.25$

14.(0.01)

If final pressure of gas in tube is P_2 then :



$$P_2 - \frac{2T}{r} = P_0$$

(as levels inside and outside are same)

i.e. $P_2 = P_0 + \frac{2T}{r}$

but as temperature remains constant $\therefore$ $P_1V_1 = P_2V_2$

$$P_0AL = \left(P_0 + \frac{2T}{r}\right)A(L-x)$$

$$\Rightarrow \quad (1.01\times10^5)(0.11) = \left(1.01\times10^5 + \frac{2\times5.05\times10^{-2}}{10^{-5}}\right)(0.11-x); \text{ On solving} \Rightarrow \boxed{x = 0.01m}$$

15.(0.44) Let the distance between the plates be d. Then the electric field $E = \frac{V_0}{d}\sin(2\pi vt)$. The coduction current density is given by the ohm's law = J = σE . $\Rightarrow$ $J^c = \frac{1}{\rho}\frac{V_0}{d}\sin(2\pi vt) = \frac{V_0}{\rho d}\sin(2\pi vt)$

Whre, $J_0^c = \frac{V_0}{\rho d}$.

The displacement current density is given as $J^d = \varepsilon\frac{\partial E}{dt} = \varepsilon\frac{\partial}{dt}\left\{\frac{V_0}{d}\sin(2\pi vt)\right\} = \frac{\varepsilon 2\pi_v V_0}{d}\cos(2\pi vt)$

$= J_0^d \cos(2\pi vt)$, *where* $J_0^d = \frac{2\pi v\varepsilon V_0}{d}$

$$\frac{J_0^d}{J_0^c} = \frac{2\pi v\varepsilon V_0}{d}\cdot\frac{\rho d}{V_0} = 2\pi v\varepsilon\rho = 2\pi\times80\,\varepsilon_0 v\times0.25 = 4\pi\varepsilon_0 v\times10 = \frac{10v}{9\times10^9} = \frac{4}{9}$$

16.(600) Let the initial temperature, pressure and volume on both sides be T_0, P_0 and V_0

The final pressure on both sides will be the same. Let this pressure be P

The final volume on the two sides is $1.1V_0$ and $0.9V_0$

The process on the right side is adiabatic. So, $P_0V_0^{5/3} = P(0.9V_0)^{5/3} \Rightarrow P = 1.2P_0$

For the right compartment, work done by the gas

$$W_2 = -\Delta U_2 = -\left[\frac{3}{2}(1.2P_0)(0.9V_0) - \frac{3}{2}P_0V_0\right] = -0.12P_0V_0$$

Also, work done by the gas in the left compartment, $W_1 = -W_2 = 0.12P_0V_0$

Change in internal energy of the gas in the left compartment,

$$\Delta U_1 = U_f - U_i = \frac{3}{2}(1.2P_0)(1.1V_0) - \frac{3}{2}P_0V_0 = 0.48P_0V_0$$

So, heat added, $\Delta Q = \Delta U_1 + W_1 = 0.60P_0V_0$

Hence, $\Delta Q = (0.60)(10^4)(0.1) = 600J$

17.(160) $f_1 \pm f_2 = 3.5$ (since $f_2 > f_1$ $\because$ on decreasing f_1, their difference increases.)

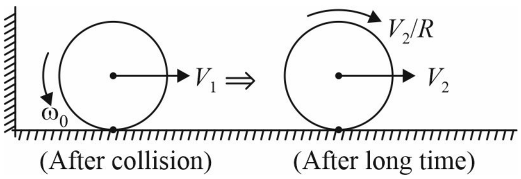
$\Rightarrow \quad f_2 - f_1 = 3.5$

$f_3 = f_2\left(\frac{332+5}{332-5}\right) = f_2\left(\frac{337}{327}\right) \Rightarrow \quad f_2\left(\frac{337}{327}\right) - f_2 = 5 \Rightarrow f_2\left(\frac{10}{327}\right) = 5 \Rightarrow f_2 = \frac{327}{2} = 163.5\text{ Hz}$

$f_1 = 160 Hz$

18.(4) Let V_1 just after collision and V_2 after long time [ω will not change during collision]

$e = \frac{V_1}{V} \Rightarrow V_1 = 0.7 \times 20 = 14\text{ m/s}$



Angular momentum conservation about O

$mV_1R - I\omega_0 = mV_2R + I\frac{V_2}{R}$

Putting values.

$V_2 = 4$ m/s

CHEMISTRY

1.(A) $XY \rightleftharpoons X^+ + Y^-$ $\qquad K_1 = \dfrac{[X^+][Y^-]}{[XY]}$

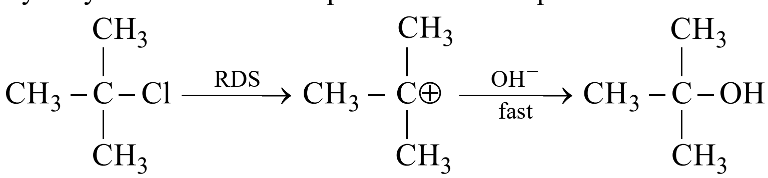
$XY + Y^- \rightleftharpoons XY_2^-$ $\qquad K_2 = \dfrac{[XY_2^-]}{[XY][Y^-]}$

$$\frac{K_1}{K_2} = \frac{[X^+][Y^-][XY][Y^-]}{[XY_2^-][XY]} = \frac{[X^+][Y^-]^2}{[XY_2^-]} \quad \Rightarrow \quad \frac{[X^+]}{[XY_2^-]} = \frac{K_1}{K_2}\frac{1}{[Y^-]^2}$$

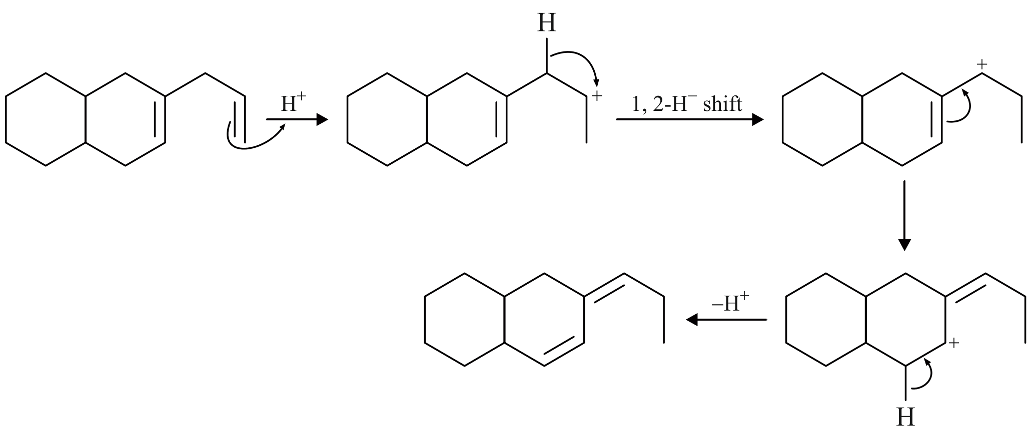
2.(B) $\mu = 0$ because of planar symmetrical structure.

3.(B) Due to strong oxidising character of Pb^{4+} and reducing character of I^- ions, PbI_4 do not exist.

4.(B) Hydrolysis of 3° RX is independent of nucleophile concentration (S_N1 reaction).



5.(C)



6.(B) There are 4α H-atom.

7.(BC) Statement-A and Statement-D are incorrect.

Statement-A

$$P(s) + \frac{3}{2}Cl_2(g) \longrightarrow PCl_3(g) \qquad \Delta_f H = 300\,kJ$$

$$\Delta H_r = \sum(BE)_{reactant} - \sum(BE)_{product}$$

$$300 = (320 + 3\times 120) - 3\,B.E.(P-Cl) \qquad \therefore \qquad B.E.\text{ of } P-Cl = 126.6\,kJ/mole$$

Statement-B

$$\Delta_r H = (B.E.)_R - (B.E.)_P$$

$$= \left[3(C-H) + (C-O) + (O-H) + (H-Cl)\right] - \left[3\times(C-H) + (C-Cl) + 2(O-H)\right]$$

$$= \left[3\times 400 + 330 + 450 + 430\right] - \left[3\times 400 + 320 + 2\times 480\right] = -10\,kJ/mole$$

Statement-C

$$N_2(g) + 2H_2(g) \longrightarrow N_2H_4(g)$$

$$\Delta_f H(N_2H_4,\ g) = \left[B.E.(N\equiv N) + 2\,B.E.\text{ of } H-H\right] - \left[B.E.(N-N) + 4\,B.E.\text{ of } N-H\right]$$

$$= (940 + 2\times 430) - (160 + 4\times 400) = 1800 - 1760 = 40\,kJ/mole$$

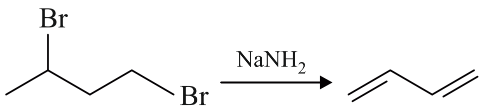
Statement-D

B.E. of C – H bond cannot be determined.
Since B.E. of (C – C) is not given.

8.(BC) $CaC_2 + H_2O \longrightarrow H-C \equiv C-H + Ca(OH)_2$

$CH_3C \equiv CMgI + H_2O \longrightarrow CH_3C \equiv CH + MgIOH$

$Al_4C_3 + H_2O \longrightarrow Al(OH)_3 + CH_4$



9.(B)

10.(ACD)

$[Cu(CN)_4]^{3-} \to sp^3$

$[Cu(Py)_4]^{+} \to sp^3$

$[Ni(CO)_4] \to sp^3$

$[Ni(CN)_4]^{2-} \to dsp^2$

$[Co(CO)_4]^{-} \to sp^3$

$[CoCl_4]^{2-} \to sp^3$

$[Fe(CO)_4]^{2-} \to sp^3$

$[FeCl_4]^{-} \to sp^3$

11.(ABCD)

(A) PbO dissolves in HNO_3, because $Pb(NO_3)_2$ formed is soluble in H_2O. If H_2SO_4 or HCl is used, insoluble layer of $PbSO_4$ or $PbCl_2$ is deposited on the surface and prevent the further reaction.

(B) $\overset{+4}{Pb}O_2 + 4HCl \longrightarrow \overset{+2}{Pb}Cl_2 + Cl_2 + 2H_2O$

$\overset{+4}{Pb}O_2 + SO_2 \longrightarrow \overset{+2}{Pb}SO_4$

Pb^{4+} change to Pb^{2+}, reaction occurs.

(C) Pb_3O_4 is mixture of PbO and PbO_2

PbO_2 is an oxidizing agent, $PbCl_2$ and Cl_2 are formed

$Pb_3O_4 \longrightarrow 3PbO + PbO_2$; $4HCl + PbO_2 \longrightarrow PbCl_2 + Cl_2 + H_2O$

(D) Sn^{2+} is a reducing agent and Fe^{3+} is an oxidising agent.
Thus they cannot exist in same solution.

12.(AB)

(A) For both urea and glucose $i = 1$

(B) For both $K_4[Fe(CN)_6]$ and $Al_2(SO_4)_3$ $i = 1 + 4\alpha$

(C) For association of solute in a solution $i < 1$

(D) Glucose and sucrose do not undergo either association or dissociation

13.(5) $HClO_4$, HCl, CH_3COOH, $\underset{\text{Both salt of WB/SA}}{\underline{(NH_4)_2SO_4 \text{ and } NH_4Cl}}$

14.(5) Energy released = Energy due to formation of two single bond.

$= 2 \times 331 = 662\,\text{kJ/mole}$ of propene

$\Delta H_{\text{Polymerisation/mole}} = 590 - 662 = -72\,\text{kJ/mole}$

$\Delta H_{\text{Polymerisation}} = -72 \times n = -360 \quad \Rightarrow \quad n = 5$

15.(315.36)

Moles of H_2O_2 = Moles of $H_2 = \frac{100}{34}$

Weight of $H_2 = \frac{100}{34} \times 2 = \frac{100}{17} = zit \times \frac{50}{100}$

$$\frac{100}{17} = \frac{1 \times i \times 0.5 \times 3600}{96500} \quad \Rightarrow \quad i = 315.36\,\text{A}$$

16.(2) **First part**

mmole of $Na_2S_2O_3$ used $= 8 \times 2 = 16$

$$I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$$

1 mole 2 mole

mmole of I_2 used $= \frac{1}{2}$ mmole of $Na_2S_2O_3 = \frac{16}{2} = 8$

Second part

$$3I_2 + 6NaOH \longrightarrow 5NaI + NaIO_3 + 3H_2O$$

mmole of H_2SO_4 = excess NaOH $= 30 \times 0.1$

Excess mmole of NaOH $= 2 \times$ mmole of $H_2SO_4 = 2 \times 30 \times 0.1 = 6$

mmole of NaOH used $= 30 - 6 = 24$

mmole of I_2 used $= \frac{1}{2}$ mmole of NaOH used $= \frac{24}{2} = 12$ mmole of I_2 used

Total mmole of I_2 = Part - 1 + Part - II $= 8 + 12 = 20$

Molarity of $I_2 = \frac{\text{mmole}}{V\,\text{mL}} = \frac{20}{200} = 0.1\text{M}$

20 times the initial $M_{I_2} = 0.1 \times 20 = 2$

17.(4.77)

$$\frac{(r_2)_{95}}{(r_1)_{95}} = \left(\frac{\mu_2}{\mu_1}\right)^{\frac{\Delta T}{10}} = \left(\frac{2.5}{2}\right)^7 = 4.768 \simeq 4.77$$

18.(5) 5 compounds (2, 3, 5, 6 and 7) are more basic than aniline.

MATHEMATICS

1.(A) The circle is $x^2+y^2=\dfrac{8}{5}$.

2.(A) $\left(\dfrac{\alpha-\gamma}{\beta-\gamma}\right)\times\left(\dfrac{\alpha-\delta}{\beta-\delta}\right)=\dfrac{\alpha^2-\alpha(\gamma+\delta)+\gamma\delta}{\beta^2-\beta(\gamma+\delta)+\gamma\delta}=\dfrac{\alpha^2-\alpha(-p)-r}{\beta^2-\beta(-p)-r}=\dfrac{\alpha^2+p\alpha-r}{\beta^2+p\beta-r}=\dfrac{-q-r}{-q-r}=1$

3.(B) $f(x)=\max\left\{\cos^{-1}\cos x,\ |x-\pi|\right\}$

$$\Rightarrow\ f(x)=\begin{cases}\pi-x & 0<x<\pi/2\\ x & \pi/2\le x\le\pi\\ 2\pi-x & \pi\le x\le 3\pi/2\\ x-\pi & 3\pi/2\le x\le 2\pi\end{cases}$$

Area $=4\cdot\dfrac{1}{2}\left(\pi+\dfrac{\pi}{2}\right)\cdot\dfrac{\pi}{2}=\dfrac{3\pi^2}{2}$ *sq. units.*

4.(C) The expression $ax^4+bx^3+(a+1)x^2+bx+1$

$=\left(x^2+1\right)\left(ax^2+bx+1\right)$

$\therefore\ \left(x^2+1\right)$ is positive $\forall\ x\in R$

For ax^2+bx+c to be positive $\forall\ x\in R$ $a>0$ and $b^2-4c<0$

If $b=1,a$ can take 9 value from 1 to 9.

$b=2,a$ can take 8 values from 2 to 9.

$b=3,a$ can take 7 value from 3 to 9.

$b=4,a$ can take 5 value from 5 to 9.

$b=5,a$ can take 3 value from 7 to 9.

b can not take value 6, 7, 8, 9

No. of favourable cases = 9 + 8 +7 + 5 + 3 = 32

Total no. of exhaustive cases = 9 × 9 = 81

Required Probability $=\dfrac{32}{81}$

5.(C) Onto: $\forall x\in X,\ \exists x\in X$ such that $g(g(x))=x$

One-one: $g(x_1)=g(x_2)$

$\Rightarrow\quad g(g(x_1))=g(g(x_2))$

$\Rightarrow\quad x_1=x_2$

6.(A) Area $=2\displaystyle\int_0^2\left\{\frac{\pi}{10}\left(x^2+1\right)-\sin^{-1}(x+1)\right\}dx$

Area $=\dfrac{14}{15}\pi=\dfrac{p}{q}\pi$

So, p = 14 and q = 15

$q-p=15-14=1$

7.(AC) Any point on line L is $(3k+2,\ 4k+1, 5k+6)$.

$\therefore \quad K=0 \therefore A \equiv (2,1,6)$

Image $A' \equiv (x_2\ y_2\ z_2) \Rightarrow \dfrac{x_2-2}{1} = \dfrac{y_2-1}{1} = \dfrac{z_2-6}{-2} = 4$

$\therefore \quad A' \equiv (6,5,-2)$

Putting $(3k+2, 4k+1, 5k+6)$ in equation of plane, we get $B \equiv (-10,-15,-14)$.

Equation of reflected ray BA' is $\dfrac{x+10}{4} = \dfrac{y+15}{5} = \dfrac{z+14}{3}$

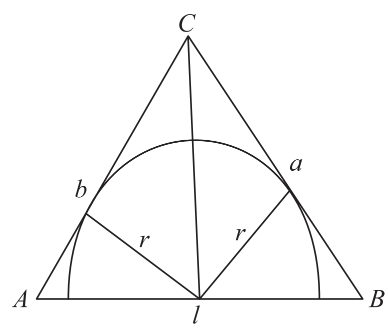
8.(AB) $|z_3| = \dfrac{|z_2||z_1-z_4|}{|\bar{z}_1 z_4 - 1|} = \dfrac{|z_1-z_4|}{|\bar{z}_1 z_4 - 1|} \le 1$

$|z_1 - z_4| \le |\bar{z}_1 z_4 - 1|$

$\Rightarrow |z_1-z_4|^2 \le |\bar{z}_1 z_4 - 1|^2 \qquad \Rightarrow |z_1|^2 + |z_4|^2 - |z_1|^2|z_4|^2 - 1 \le 0$

$\Rightarrow \left(|z_1|^2 - 1\right)\left(|z_4|^2 - 1\right) \ge 0 \qquad \Rightarrow |\ z_4| \ge 1$

9.(AC) $\dfrac{1}{2}ra + \dfrac{1}{2}rb = \dfrac{1}{2}ab\sin c$



$r = \dfrac{2\Delta}{a+b}$

$\therefore\ r = \dfrac{2abc}{4R(2R\sin A + 2R\sin B)} = \dfrac{abc}{4R^2(\sin A + \sin B)}$

Also, $x = \dfrac{2ab}{a+b}\cos\left(\dfrac{c}{2}\right)$

$r = \dfrac{2\times\frac{1}{2}ab\sin c}{a+b} = \dfrac{2ab\sin\frac{c}{2}\cos\frac{c}{2}}{a+b} = \dfrac{2ab\cos\frac{c}{2}}{a+b}\sin\dfrac{c}{2} = x\sin\left(\dfrac{c}{2}\right)$

10.(BCD) $x > \sin x$ in $\left[0, \dfrac{\pi}{2}\right]$ $\quad \therefore \cos x < \cos(\sin x) \Rightarrow I_3 < I_1$

and $\sin(\cos x) < \cos x \Rightarrow I_2 < I_3$

11.(CD) $\left(PQP^T\right)^T = PQ^T P \ne PQP^T$ is not symmetric matrix.

$(PQ - QP)^T = Q^T P^T - P^T Q^T$

$Q = |P|\dfrac{adj\ P}{|P|} \Rightarrow Q = adj(p)$

$\Rightarrow \left(adj\left(P^T\right) - Q\right)^T = \left(adj\left(P^T\right) - adj\ p\right)^T = adj(p) - adj(P^T)$

and $Q = -P^T \Rightarrow Q = p \Rightarrow Q$ is also skew symmetric matrix.

12.(ABD) $f(x)=\begin{cases}\left\{\cos\left(\frac{1}{x}\right)\right\}\times\left\{\ln^2(1+x)\right\} & x>0\\ 0 & x\le 0\end{cases}$

$$\lim_{x\to0^+} f(x)=\lim_{x\to0^+} x^2\cos\left(\frac{1}{x}\right)\frac{\ln^2(1+x)}{x^2}=0$$

$$f'(0)=\lim_{x\to0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0} x\cos\left(\frac{1}{x}\right)\frac{\ln^2(1+x)}{x^2}=0$$

$$f'(x)=\begin{cases}\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\ln^2(1+x)+\cos\left(\frac{1}{x}\right)\frac{2\ln(1+x)}{1+x} & x>0\\ 0 & x\le 0\end{cases}$$

$f'(x)$ is not continuous at $x=0$

13.(1.60) $f(x)=(x-2)^{-5}+(x-2)^{-4}+3(x-2)^{-3}+1+(x-2)^8+(x-2)^{10}$

Say $(x-2)=t.(\in>0)$

$f(x)=t^{-5}+t^{-4}+3t^{-3}+1+t^8+t^{10}$

Now Apply $AM\ge G.M.$

$$\frac{t^{-5}+t^{-4}+t^{-3}+t^{-3}+t^{-3}+1+t^8+t^{10}}{8}\ge(1)^{\frac{1}{8}}$$

$\Rightarrow\quad f(x)\ge 8\Rightarrow$ minimum value of $f(x)$ is 8 $\Rightarrow\quad A=8\Rightarrow\quad \frac{A}{5}=\frac{8}{5}=1.60$

14.(3.20) $(\vec a\times\vec b)\cdot(\vec b\times\vec c)=(\vec b\cdot\vec c)\cdot(\vec a\cdot\vec b)-\vec a\cdot\vec c$

Given that $|\vec a+\vec b+\vec c|=\sqrt3\Rightarrow\vec a\cdot\vec b+\vec b\cdot\vec c+\vec c\cdot\vec a=0$

$\lambda=(\vec a\cdot\vec b)(\vec b\cdot\vec c)+(\vec b\cdot\vec c)(\vec c\cdot\vec a)+(\vec c\cdot\vec a)(\vec a\cdot\vec b)-\vec a\cdot\vec b-\vec b\cdot\vec c-\vec c\cdot\vec a$

$\lambda=(\vec a\cdot\vec b)(\vec b\cdot\vec c)+(\vec b\cdot\vec c)(\vec c\cdot\vec a)+(\vec c\cdot\vec a)(\vec a\cdot\vec b)$

$\Rightarrow\lambda\le0\left[\text{since } x+y+z=0,\ xy+yz+zx\le0\right]$

$\lambda_{\max}=0$ only when $\vec a\cdot\vec b=\vec b\cdot\vec c=\vec c\cdot\vec a=0$

$\Rightarrow\quad \vec a\perp\vec b,\ \vec b\perp\vec c$ and $\vec c\perp\vec a$

$(2\vec a+3\vec b+4\vec c)\cdot(\vec a\times\vec b+5\vec b\times\vec c+6\vec c\times\vec a)=32=K$

15.(1.20) $=\lim_{x\to1^-}\frac{1-\cos(a\cos^{-1}x)}{1-x^2}\begin{bmatrix}\text{put }\theta=\cos^{-1}x\\ x=\cos\theta\end{bmatrix}=\lim_{\theta\to0^+}\frac{1-\cos(a\theta)}{1-\cos^2\theta}\left(\frac{0}{0}\text{ form}\right)=\lim_{\theta\to0^+}\frac{2\sin^2\left(\frac{a\theta}{2}\right)}{\sin^2\theta}$

$$=\lim_{\theta\to0^+}\frac{2\left(\frac{a\theta}{2}\right)^2}{\frac{\sin^2\theta}{\theta^2}\times\theta^2}=\frac{a^2}{2}=18\text{ (given)}\ \Rightarrow\ a=\pm6\text{ then }\frac{|a|}{5}=\frac{6}{5}=1.2$$

16.(2.50) $2f(x)+xf\left(\frac{1}{x}\right)-2f\left\{\left|\sqrt2\sin\left[\pi\left(x-\frac{1}{4}\right)\right]\right|\right\}=4\cos^2\frac{\pi x}{2}+x\cos\frac{\pi}{x}$

Put $x = 1$ $\quad 2f(1) + f(1) - 2f(1) = -1 \quad \Rightarrow \quad (1) = -1$

Put $x = 2$ $\quad 2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4$

Put $x = \frac{1}{2}$ $\quad 2f\left(\frac{1}{2}\right) + \frac{1}{2}f(2) - 2f(1) = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$\frac{\cdot 3}{2}f(2) = \frac{3}{2} \Rightarrow f(2) = 1$

and $f\left(\frac{1}{2}\right) = 0$

So, $\dfrac{f\left(\frac{1}{2}\right) + f(2) + f(1) + 5}{2} = \dfrac{0 + 1 + (-1) + 5}{2} = 2.50$

17.(11) $f(x) = x^2 + x + \frac{3}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{1}{2} \geq \frac{1}{2}$

$g(f(x)) = (f(x))^2 + a(f(x)) + 1$

for $g(f(x)) = 0$ have solution,

$(f(x))^2 + a(f(x)) + 1 = 0$

$\Rightarrow a = -\left[f(x) + \frac{1}{f(x)}\right] \leq -2$,

because $f(x)$ is positive and equality holds when $f(x) = 1$, which is in the range of $f(x)$ so for no solution of $g(f(x)) = 0$, $a > -2$.

18.(3.00) Let us add one more number a_{n+1}, to sequence. The number a_{n+1} is such that $|a_{n+1}| = |a_n + 1|$.

Squaring all the numbers we have

$a_1^2 = 0$

$a_2^2 = a_1^2 + 2a_1 + 1$

$a_3^2 = a_2^2 + 2a_2 + 1$

$a_{n+1}^2 = a_n^2 + 2a_n + 1$

Adding above inequality, we have

$a_1^2 + a_2^2 - - + a_{n+1}^2 = a_1^2 + a_2^2 - - + a_n^2 + 2(a_1 + a_2 + - - + a_n) + n$

$2(a_1 + a_2 + a_3 - - + a_n) = -n + a_{n+1}^2 \geq -n$

$\dfrac{a_1 + a_2 + a_3 - - - - + a_n}{n} \geq \dfrac{-1}{2} = \dfrac{-p}{q}$

then $(p + q) = (1 + 2) = 3$